

### 5.6.1. Quantifier Construction Problems

A. For each of the strings of symbols below, decide whether it a **formula** of the formal language according to the construction rules listed here.

#### Construction Rules (*Final Version*)

##### Atomic Formulas:

- A1. Sentence letters are atomic formulas
- A2. A predicate letter followed by a name letter *or* variable is an atomic formula.

##### Formulas:

- 1. Atomic formulas are formulas.
- 2. If  $\bullet$  is a formula, then  $\sim\bullet$  is a formula.
- 3. If  $\bullet$  and  $\blacktriangle$  are formulas, then  $(\bullet \wedge \blacktriangle)$  is a formula.
- 4. If  $\bullet$  and  $\blacktriangle$  are formulas, then  $(\bullet \vee \blacktriangle)$  is a formula.
- 5. If  $\bullet$  and  $\blacktriangle$  are formulas, then  $(\bullet \rightarrow \blacktriangle)$  is a formula.
- 6. If  $\bullet$  and  $\blacktriangle$  are formulas, then  $(\bullet \leftrightarrow \blacktriangle)$  is a formula.
- 7. If  $\star$  is a variable and  $\bullet$  is a formula, then

$\exists\star\bullet$

and

$\forall\star\bullet$

are both formulas.

- |   |   |
|---|---|
| 1. $(\forall x Gx \wedge \forall x Hx)$ | 2. $HAB$                                |
| 3. $\forall x (Gx \wedge Hx)$           | 4. $(\forall x Gx \wedge Hx)$           |
| 5. $(A \wedge P)$                       | 6. $(P \rightarrow MC)$                 |
| 7. $(\forall x GA \wedge HB)$           | 8. $\forall x (GA \wedge Hx)$           |
| 9. $\forall A (GA \wedge HA)$           | 10. $(\forall x) (GA \wedge Hx)$        |
| 11. $\forall x P$                       | 12. $(\forall x Q \wedge \forall x Hy)$ |

**B.** For each of the genuine formulas in A (repeated below), state whether it is a **sentence** or a **quasi-sentence**.

- |   |   |
|---|---|
| 1. $(\forall x Gx \wedge \forall x Hx)$ | 2. $HAB$                                |
| 3. $\forall x (Gx \wedge Hx)$           | 4. $(\forall x Gx \wedge Hx)$           |
| 5. $(A \wedge P)$                       | 6. $(P \rightarrow MC)$                 |
| 7. $(\forall x GA \wedge HB)$           | 8. $\forall x (GA \wedge Hx)$           |
| 9. $\forall A (GA \wedge HA)$           | 10. $(\forall x)(GA \wedge Hx)$         |
| 11. $\forall x P$                       | 12. $(\forall x Q \wedge \forall x Hy)$ |

**C.** For each of the formulas in the above list featuring a quantifier, state whether or not its quantifier is **vacuous**.